(b) $x, x-1, x^2+1, x^2-1$

(c) $x^2, x^2-x-1, x+1$ (d) $2x, x-2$

15. Let S be the subspace of P_3 consisting of all polynomials $p(x)$ such that $p(0)=0$, and let T be the subspace of all polynomials $q(x)$ such that $q(1)=0$. Find bases for

(a) S (b) T (c) $S \cap T$

16. In $\mathbb{R}^4$, let U be the subspace of all vectors of the form $(u_1, u_2, 0, 0)^T$, and let V be the subspace of all vectors of the form $(0, v_2, v_3, 0)^T$. What are the dimensions of $U, V, U \cap V, U+V$? Find a basis for each of these four subspaces. (See Exercises 20 and 22 of Section 2.)

17. Is it possible to find a pair of two-dimensional subspaces U and V of $\mathbb{R}^3$ such that $U \cap V = \{\mathbf{0}\}$? Prove your answer. Give a geometrical interpretation of your conclusion. [*Hint*: Let $\{\mathbf{u}_1, \mathbf{u}_2\}$ and $\{\mathbf{v}_1, \mathbf{v}_2\}$ be bases for U and V, respectively. Show that $\mathbf{u}_1, \mathbf{u}_2, \mathbf{v}_1, \mathbf{v}_2$ are linearly dependent.]

18. Show that if U and V are subspaces of $\mathbb{R}^n$ and $U \cap V = \{\mathbf{0}\}$, then

$$\dim(U+V) = \dim U + \dim V$$