SECTION 6 EXERCISES

1. For each of the following, use the Gram–Schmidt process to find an orthonormal basis for $R(A)$:

(a) $A = \begin{pmatrix} -1 & 3 \\ 1 & 5 \end{pmatrix}$ **(b)** $A = \begin{pmatrix} 2 & 5 \\ 1 & 10 \end{pmatrix}$

2. Factor each of the matrices in Exercise 1 into a product QR, where Q is an orthogonal matrix and R is upper triangular.

3. Given the basis $\{(1, 2, -2)^T, (4, 3, 2)^T, (1, 2, 1)^T\}$ for $\mathbb{R}^3$, use the Gram–Schmidt process to obtain an orthonormal basis.

4. Consider the vector space $C[-1, 1]$ with inner product defined by

$$\langle f, g \rangle = \int_{-1}^{1} f(x)g(x)\,dx$$

Find an orthonormal basis for the subspace spanned by 1, x, and x^2.

5. Let

$$A = \begin{pmatrix} 2 & 1 \\ 1 & 1 \\ 2 & 1 \end{pmatrix} \quad \text{and} \quad \mathbf{b} = \begin{pmatrix} 12 \\ 6 \\ 18 \end{pmatrix}$$

(a) Use the Gram–Schmidt process to find an orthonormal basis for the column space of A.

(b) Factor A into a product QR, where Q has an orthonormal set of column vectors and R is upper triangular.

(c) Solve the least squares problem $A\mathbf{x} = \mathbf{b}$.