2 Exercises

In Exercises 1–6, verify Green's theorem for the given vector field

$$\mathbf{F} = M(x, y)\,\mathbf{i} + N(x, y)\,\mathbf{j}$$

and region D by calculating both

$$\oint_{\partial D} M\,dx + N\,dy \quad \textit{and} \quad \iint_D (N_x - M_y)\,dA.$$

1. $\mathbf{F} = -x^2 y\,\mathbf{i} + xy^2\,\mathbf{j}$, D is the disk $x^2 + y^2 \le 4$.

2. $\mathbf{F} = (x^2 - y)\,\mathbf{i} + (x + y^2)\,\mathbf{j}$, D is the rectangle bounded by $x = 0$, $x = 2$, $y = 0$, and $y = 1$.

3. $\mathbf{F} = y\,\mathbf{i} + x^2\,\mathbf{j}$, D is the square with vertices $(1, 1)$, $(-1, 1)$, $(-1, -1)$, and $(1, -1)$.

4. $\mathbf{F} = 2y\,\mathbf{i} + x\,\mathbf{j}$, D is the semicircular region $x^2 + y^2 \le a^2$, $y \ge 0$.

5. $\mathbf{F} = 3y\,\mathbf{i} - 4x\,\mathbf{j}$, D is the elliptical region $x^2 + 2y^2 \le 4$.

[2] For details of the type of limit argument we have in mind, see O. D. Kellogg, *Foundations of Potential Theory* (Springer, Berlin, 1929; reprinted by Dover Publications, New York, 1954), pp. 113–119, where a limit argument is given in the case of Gauss's theorem. For a proof of Green's theorem that avoids the limit argument, see D. V. Widder, *Advanced Calculus*, 2nd ed., (Prentice-Hall, Englewood Cliffs, 1961; reprinted by Dover Publications, New York, 1989), pp. 223–225.

[3] See also M. Kline, *Mathematical Thought from Ancient to Modern Times* (Oxford Press, New York, 1972), p. 683.