3 Exercises

1. Consider the line integral $\int_C z^2\,dx + 2y\,dy + xz\,dz$.
   (a) Evaluate this integral, where C is the line segment from (0, 0, 0) to (1, 1, 1).
   (b) Evaluate this integral, where C is the path from (0, 0, 0) to (1, 1, 1) parametrized by $\mathbf{x}(t) = (t, t^2, t^3)$, $0 \le t \le 1$.
   (c) Is the vector field $\mathbf{F} = z^2\,\mathbf{i} + 2y\,\mathbf{j} + xz\,\mathbf{k}$ conservative? Why or why not?

2. Let $\mathbf{F} = 2xy\,\mathbf{i} + (x^2 + z^2)\,\mathbf{j} + 2yz\,\mathbf{k}$.
   (a) Calculate $\int_C \mathbf{F} \cdot d\mathbf{s}$, where C is the path parametrized by $\mathbf{x}(t) = (t^2, t^3, t^5)$, $0 \le t \le 1$.
   (b) Calculate $\int_C \mathbf{F} \cdot d\mathbf{s}$, where C is the straight-line path from (0, 0, 0) to (1, 0, 0), followed by the straight-line path from (1, 0, 0) to (1, 1, 1).
   (c) Does $\mathbf{F}$ have path-independent line integrals? Explain your answer.

In Exercises 3–17, determine whether the given vector field $\mathbf{F}$ *is conservative. If it is, find a scalar potential function for* $\mathbf{F}$.

3. $\mathbf{F} = e^{x+y}\,\mathbf{i} + e^{xy}\,\mathbf{j}$

4. $\mathbf{F} = 2x \sin y\,\mathbf{i} + x^2 \cos y\,\mathbf{j}$

5. $$\mathbf{F} = \left(3x^2 \cos y + \frac{y}{1 + x^2y^2}\right)\mathbf{i} + \left(x^3 \sin y + \frac{x}{1 + x^2y^2}\right)\mathbf{j}$$

6. $$\mathbf{F} = \frac{xy^2}{(1 + x^2)^2}\,\mathbf{i} + \frac{x^2y}{1 + x^2}\,\mathbf{j}$$

7. $\mathbf{F} = (e^{-y} - y \sin xy)\,\mathbf{i} - (xe^{-y} + x \sin xy)\,\mathbf{j}$

8. $\mathbf{F} = (6xy^2 + 2y^3)\,\mathbf{i} + (6x^2y - xy)\,\mathbf{j}$

9. $\mathbf{F} = (6xy^2 - 3x^2)\,\mathbf{i} + (y^2 + 6x^2y)\,\mathbf{j}$

10. $\mathbf{F} = (xyz^3 + xy - z^2)\,\mathbf{i} + (2x^2z^3 - y^2 + 2yz)\,\mathbf{j} + (6x^2y - y^2z)\,\mathbf{k}$

11. $\mathbf{F} = (4xyz^3 - 2xy)\,\mathbf{i} + (2x^2z^3 - x^2 + 2yz)\,\mathbf{j} + (6x^2yz^2 + y^2)\,\mathbf{k}$

12. $\mathbf{F} = (2xz - y^2 + yze^{xyz})\,\mathbf{i} - (2xy + xze^{xyz})\,\mathbf{j} + (x^2 + xye^{xyz})\,\mathbf{k}$

13. $\mathbf{F} = (2x + y)\,\mathbf{i} + (z \cos yz + x)\,\mathbf{j} + (y \cos yz)\,\mathbf{k}$

14. $\mathbf{F} = (y + z)\,\mathbf{i} + 2z\,\mathbf{j} + (x + y)\,\mathbf{k}$

15. $\mathbf{F} = e^x \sin y\,\mathbf{i} + e^x \cos y\,\mathbf{j} + (3z^2 + 2)\,\mathbf{k}$

16. $$\mathbf{F} = 3x^2\,\mathbf{i} + \frac{z^2}{y}\,\mathbf{j} + 2z \ln y\,\mathbf{k}$$

17. $\mathbf{F} = (e^{-yz} - yze^{xyz})\,\mathbf{i} + xz(e^{-yz} + e^{xyz})\,\mathbf{j} + xy(e^{-yz} - e^{xyz})\,\mathbf{k}$

18. Of the two vector fields
$$\mathbf{F} = xy^2z^3\,\mathbf{i} + 2x^2y\,\mathbf{j} + 3x^2y^2z^2\,\mathbf{k}$$
and
$$\mathbf{G} = 2xy\,\mathbf{i} + (x^2 + 2yz)\,\mathbf{j} + y^2\,\mathbf{k},$$
one is conservative and one is not. Determine which is which, and, for the conservative field, find a scalar potential function.

19. (a) Let f be a function of class C^1 defined on a connected domain in $\mathbf{R}^n$. Show that if the gradient of f vanishes at all $\mathbf{x} = (x_1, \ldots, x_n)$ in its domain, then f is constant.