## 2 Exercises

**1.** Let $\mathbf{X}(s,t) = (s, s+t, t)$, $0 \le s \le 1$, $0 \le t \le 2$. Find

$$\iint_{\mathbf{X}} (x^2 + y^2 + z^2)\, dS.$$

**2.** Let $D = \{(s,t) \mid s^2 + t^2 \le 1,\ s \ge 0,\ t \ge 0\}$ and let $\mathbf{X}\colon D \to \mathbf{R}^3$ be defined by $\mathbf{X}(s,t) = (s+t, s-t, st)$.

(a) Determine $\iint_{\mathbf{X}} f\, dS$, where $f(x,y,z) = 4$.

(b) Find the value of $\iint_{\mathbf{X}} \mathbf{F} \cdot d\mathbf{S}$, where $\mathbf{F} = x\,\mathbf{i} + y\,\mathbf{j} + z\,\mathbf{k}$.

**3.** Find the flux of $\mathbf{F} = x\,\mathbf{i} + y\,\mathbf{j} + z\,\mathbf{k}$ across the surface $S$ consisting of the triangular region of the plane $2x - 2y + z = 2$ that is cut out by the coordinate planes. Use an upward-pointing normal to orient $S$.

**4.** This problem concerns the two surfaces given parametrically as

$$\mathbf{X}(s,t) = (s\cos t, s\sin t, 3s^2),$$
$$0 \le s \le 2,\ 0 \le t \le 2\pi.$$

and

$$\mathbf{Y}(s,t) = (2s\cos t, 2s\sin t, 12s^2),$$
$$0 \le s \le 1,\ 0 \le t \le 4\pi.$$

(a) Show that the images of $\mathbf{X}$ and $\mathbf{Y}$ are the same. (Hint: Give equations in $x$, $y$, and $z$ for the surfaces in $\mathbf{R}^3$ parametrized by $\mathbf{X}$ and $\mathbf{Y}$.)

(b) Calculate $\iint_{\mathbf{X}} (y\,\mathbf{i} - x\,\mathbf{j} + z^2\,\mathbf{k}) \cdot d\mathbf{S}$ and $\iint_{\mathbf{Y}} (y\,\mathbf{i} - x\,\mathbf{j} + z^2\,\mathbf{k}) \cdot d\mathbf{S}$. Reconcile your answers.

**5.** Find $\iint_S x^2\, dS$, where $S$ is the surface of the cube $[-2,2] \times [-2,2] \times [-2,2]$.

**6.** Find $\iint_S (x^2 + y^2)\, dS$, where $S$ is the lateral surface of the cylinder of radius $a$ and height $h$ whose axis is the $z$-axis.

**7.** Let $S$ be a sphere of radius $a$.

(a) Find $\iint_S (x^2 + y^2 + z^2)\, dS$.

(b) Use symmetry and part (a) to easily find $\iint_S y^2\, dS$.