8. Let S denote the sphere $x^2 + y^2 + z^2 = a^2$.

(a) Use symmetry considerations to evaluate $\iint_S x\,dS$ *without* resorting to parametrizing the sphere.

(b) Let $\mathbf{F} = \mathbf{i} + \mathbf{j} + \mathbf{k}$. Use symmetry to determine $\iint_S \mathbf{F} \cdot d\mathbf{S}$ without parametrizing the sphere.

9. Let S denote the surface of the cylinder $x^2 + y^2 = 4$, $-2 \le z \le 2$, and consider the surface integral

$$\iint_S (z - x^2 - y^2)\,dS.$$

(a) Use an appropriate parametrization of S to calculate the value of the integral.

(b) Now use geometry and symmetry to evaluate the integral without resorting to a parametrization of the surface.

In Exercises 10–18, let S denote the closed cylinder with bottom given by $z = 0$, top given by $z = 4$, and lateral surface given by the equation $x^2 + y^2 = 9$. Orient S with outward normals. Determine the indicated scalar and vector surface integrals.

10. $\iint_S z\,dS$

11. $\iint_S y\,dS$

12. $\iint_S xyz\,dS$

13. $\iint_S x^2 dS$

14. $\iint_S (x\,\mathbf{i} + y\,\mathbf{j}) \cdot d\mathbf{S}$

15. $\iint_S z\,\mathbf{k} \cdot d\mathbf{S}$

16. $\iint_S y^3\,\mathbf{i} \cdot d\mathbf{S}$

17. $\iint_S (-y\,\mathbf{i} + x\,\mathbf{j}) \cdot d\mathbf{S}$

18. $\iint_S x^2\,\mathbf{i} \cdot d\mathbf{S}$

In Exercises 19–22, find the flux of the given vector field $\mathbf{F}$ across the upper hemisphere $x^2 + y^2 + z^2 = a^2$, $z \ge 0$. Orient the hemisphere with an upward-pointing normal.

19. $\mathbf{F} = y\,\mathbf{j}$

20. $\mathbf{F} = y\,\mathbf{i} - x\,\mathbf{j}$

21. $\mathbf{F} = -y\,\mathbf{i} + x\,\mathbf{j} - \mathbf{k}$

22. $\mathbf{F} = x^2\mathbf{i} + xy\,\mathbf{j} + xz\,\mathbf{k}$

23. Let S be the parametrized helicoid $\mathbf{X}(s,t) = (s\cos t, s\sin t, t)$, with $0 \le s \le 2, 0 \le t \le 2\pi$. Determine the flux of $\mathbf{F} = y\,\mathbf{i} + x\,\mathbf{j} + z^3\,\mathbf{k}$ across S.

24. Let $\mathbf{F} = 2x\,\mathbf{i} + 2y\,\mathbf{j} + z^2\,\mathbf{k}$. Find $\iint_S \mathbf{F} \cdot d\mathbf{S}$, where S is the portion of the cone $x^2 + y^2 = z^2$ between the planes $z = -2$, and $z = 1$, oriented with outward-pointing normal.

25. Find the flux of $\mathbf{F} = y^3 z\,\mathbf{i} - xy\,\mathbf{j} + (x + y + z)\,\mathbf{k}$ across the portion of the surface $z = ye^x$ lying over the unit square $[0, 1] \times [0, 1]$ in the xy-plane, oriented by upward normal.

26. Let S denote the tetrahedron with vertices $(0, 0, 0)$, $(1, 0, 0)$, $(0, 2, 0)$, $(0, 0, 3)$ oriented by outward normal, and let $\mathbf{F} = x^2\,\mathbf{i} + 4z\,\mathbf{j} + (y - x)\,\mathbf{k}$. Find the flux of $\mathbf{F}$ across S.

27. Let S be the funnel-shaped surface defined by $x^2 + y^2 = z^2$ for $1 \le z \le 9$ and $x^2 + y^2 = 1$ for $0 \le z \le 1$.

(a) Sketch S.

(b) Determine outward-pointing unit normal vectors to S.

(c) Evaluate $\iint_S \mathbf{F} \cdot d\mathbf{S}$, where $\mathbf{F} = -y\,\mathbf{i} + x\,\mathbf{j} + z\,\mathbf{k}$ and S is oriented by outward normals.

28. The glass dome of a futuristic greenhouse is shaped like the surface $z = 8 - 2x^2 - 2y^2$. The greenhouse has a flat dirt floor at $z = 0$. Suppose that the temperature T, at points in and around the greenhouse, varies as

$$T(x, y, z) = x^2 + y^2 + 3(z - 2)^2.$$

Then the temperature gives rise to a **heat flux density field H** given by $\mathbf{H} = -k\nabla T$. (Here k is a positive constant that depends on the insulating properties of the particular medium.) Find the total heat flux outward across the dome and the surface of the ground if $k = 1$ on the glass and $k = 3$ on the ground.

29. The surface given by $\mathbf{X}(s,t) = (x(s,t), y(s,t), z(s,t))$, where

$$\begin{cases} x = \left(a + \cos\frac{s}{2}\sin t - \sin\frac{s}{2}\sin 2t\right)\cos s \\ y = \left(a + \cos\frac{s}{2}\sin t - \sin\frac{s}{2}\sin 2t\right)\sin s\ , \\ z = \sin\frac{s}{2}\sin t + \cos\frac{s}{2}\sin 2t \end{cases}$$

a is a positive constant, and $0 \le s \le 2\pi, 0 \le t \le 2\pi$, is known as a **Klein bottle**.

(a) Use a computer to plot this surface for $a = 2$.

(b) Determine (and describe) the s-coordinate curve at $t = 0$.

(c) Calculate the standard normal vector $\mathbf{N}$ along the s-coordinate curve at $t = 0$ (i.e., find $\mathbf{N}(s, 0)$). Note that $\mathbf{X}(0, 0) = \mathbf{X}(2\pi, 0)$. By comparing $\mathbf{N}(0, 0)$ and $\mathbf{N}(2\pi, 0)$, comment regarding the orientability of the Klein bottle. (See Example 8.)