

## Week in Review #8

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### Section 8.2: Expected value.

- Mean, Median, Mode
- Expected value(mean),  $E(X) = x_1 * p_1 + x_2 * p_2 + \dots + x_n p_n$ 
  - A game is said to be fair if  $E(X) = 0$
- Odds
  - Odds in favor of an event E.
  - Odds against E.
  - Given odds in favor of event E are **a** to **b**, then  $P(E) = \frac{a}{a+b}$

### Section 8.3: Variance and Standard Deviation.

- variance( var) and standard deviation (st. dev.) measure the spread of the data about the mean.
  - $\text{var} = (\text{st. dev})^2$
- Chebychev's Inequality.
- Population versus Samples
  - A population is the entire group.
    - $\mu$  is the mean of a population
    - $\sigma$  is the standard deviation of a population.
  - A sample is a subgroup of the population. This is usually used when it is difficult to get data from the entire population. This information is then used to estimate the information about the population.
    - $\bar{x}$  is the mean of the sample
    - $s_x$  is the standard deviation of the sample

1. An insurance company expects 10% of its policyholders to collect claims of \$500 this year, 40% to claim \$100, and the remaining to collect no claims. What is the expected value for the amount the company will have to pay out in claims.

2. A game consists of rolling a fair 8-sided die. The chart gives the payoff of this game. What should be charged so that this game is fair?

Die roll	8	7	5,6	1
Payoff	\$12	\$5	\$2	get your money back

3. A consumer organization evaluates new cars and reports the number of major defects( $X$ ) on each car.

X	0	1	2	3	4	5	6	7	8	9	10
cars	40	100	209	223	128	114	91	28	11	4	1

Find the following information.

- (a) mean  
 (b) median  
 (c) mode  
 (d) expected value  
 (e) variance  
 (f) standard deviation

4. The entire class of two hundred students is asked, "To the nearest hours, how many hours( $X$ ) did you study yesterday?"

X	0	1	2	3	4	5	6	7	8
students	16	28	43	43	26	22	14	6	2

Find the following information.

- (a) mean  
 (b) median  
 (c) mode  
 (d) expected value  
 (e) variance  
 (f) standard deviation

5. Two real estate companies, A and B, have offered you a job. Both companies have given you a chart showing the number of houses that a new employee might sell in a year at their firm with the associated probabilities.

Houses	16	17	18	19	20	21	22
Prob.	0.12	0.14	0.22	0.24	0.18	0.06	0.04

Houses	9	10	11	12	13	14
Prob.	0.08	0.17	0.27	0.18	0.18	0.12

The average price of a house in the locale of company A is \$98,000, whereas the average price of a house in the locale of company B is \$150,000. You have been offered a 3% commission on sales at both companies. which job offer should you accept in order to maximize your expected yearly commission and what would be your expected yearly commission?

6. If the probability of  $E$  not occurring is 0.8, what are the odds in favor of  $E$  occurring?
  
  
  
  
  
  
  
  
  
  
7. If the odds against event  $A$  are 23 to 2, what is the probability that  $A$  occurring?
  
  
  
  
  
  
  
  
  
  
8. A hair dryer has a mean lifespan of 24 months and a standard deviation of 3 months, assuming typical usage. **Estimate** the probability that one of these hair dryers will last between 20 and 28 months.
  
  
  
  
  
  
  
  
  
  
9. It has been found that with normal usage, the expected lifetime of a product is 3 years with a standard deviation of 4 months. Use this information to estimate the number of items in a shipment of 9000 that will last between 30 months and 42 months.

#### Section 8.4: The Binomial Distribution

- Binomial distribution
  - the number of trials,  $n$ , in the experiment is fixed.
  - the trials are independent of each other.
  - there are two outcomes “success” and “failure”
    - $p$  = probability of success
    - $q$  = probability of failure
    - $q = 1 - p$
  - the probability of success in each trial is fixed.
- Probability of exactly  $r$  successes in a binomial trial:  $P(x = r) = C(n, r)p^r q^{n-r}$
- binomial statistics
  - mean (expected value):  $\mu = np$
  - standard deviation:  $\sigma = \sqrt{npq}$

10. Let  $X$  be the number of successes in 5 independent trials of a binomial experiment in which the probability of success is  $\frac{2}{5}$ . Find

(a) the probability that the first three trials are success and the last two trials are not.

(b)  $P(X = 4)$

(c)  $P(2 \leq X < 5)$

11. Find the probability of at least 1 failure in five trials for a binomial distribution if  $p = \frac{3}{7}$

12. A fair die is rolled 25 times. Determine the probability of obtaining:

(a) at most 4 rolls that are fives.

(b) at least 7 rolls that are multiples of three.

(c) Find the expected number of multiples of three rolled.

- (d) Find the standard deviation of the number of multiples of three rolled.
- (e) Find the probability that at least 7 rolls are multiples of three provided the first three rolls are all 6.
13. The probability of an adverse reaction to a flu shot is 0.15. Flu shots are given to a group of 80 people.
- (a) Find the probability that exactly 5 of these people will have an adverse reaction to the shot.
- (b) Find the probability that less than 16 people will have an adverse reaction to the shot.
- (c) Find the probability that 5 to 10 people or 20 to 30 people will have an adverse reaction to the shot.
- (d) How many of these people would we expect to have an adverse reaction to the flu shot?
- (e) Find the probability that there will be 17 people with an adverse reaction to the flu shot if out of the first 10 people to get the shot exactly 5 had an adverse reaction.