Solutions to sample problems 1

5. $y - 15 = \frac{5}{11}(x - 0)$

6. (a) $C(x) = 8x + 48,000$
   (b) \$40
   (c) $R(x) = 40x$
   (d) $P(x) = 32x - 48,000$
   (e) 1500 items

7. (a) equilibrium price \$6
   (b) equilibrium quantity 7

8. (a) $y = -.0864x + 11.8636$
   (b) see class notes.
   (c) 10.5676 million cows
   (d) 2014
   (e) The prediction is -3.6884 million cows. Note: negative answers means the model has failed.
   (f) 1933

9. There is more than one answer for this problem.
$$\left[\begin{array}{ccc|c} 1 & 0 & 2 & 7 \\ 0 & 1 & 5 & 8 \end{array}\right]$$

10. There is more than one answer for this problem.
$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 8 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

11. There is more than one answer for this problem.
$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 0 & 8 \end{array}\right]$$

12. (a) **I)** x = the amount invested in high-risk stocks.
y = the amount invested in medium-risk stocks.
z = the amount invested in low-risk stocks.
II) $x + y + z = 300,000$
$.16x + .10y + .04z = 33,000$
$2x - y + 2z = 0$
III) x = \$75,000, y = \$200,000, and z = \$25,000

(b) **I)** x = number of tank cars purchased with 6,000 gallon capacity
y = number of tank cars purchased with 8,000 gallon capacity
z = number of tank cars purchased with 18,000 gallon capacity
II) x + y + z = 24
6000x + 8000y + 18000z = 250000
III) Parametric solution:
x = -29 + 5z
y =53-6z
z = any number
now to place restrictions on the parameter. Since the number of cars has to be greater than or equal to zero.

$x \geq 0$	$y \geq 0$	$z \geq 0$
$-29 + 5z \geq 0$	$53 - 6z \geq 0$	
$5z \geq 29$	$53 \geq 6z$	
$z \geq 5.8$	$8.83333 \geq z$	
	$z \leq 8.83333$	

Since the number of cars has to be less than or equal to 24.

$x \leq 24$	$y \leq 24$	$z \leq 24$
$-29 + 5z \leq 24$	$53 - 6z \leq 24$	
$5z \leq 53$	$29 \leq 6z$	
$z \leq 10.6$	$4.8333 \leq z$	
	$z \geq 4.83333$	

Now using all of the above information at the same time, we see that $5.8 \leq z \leq 8.8333$. Since we can not buy a part of a tank car, z must be an integer so the only values of z that work are 6, 7, 8.

13.
$$\left[\begin{array}{ccc|c} 3 & 6 & 15 & 9 \\ 7 & 12 & 39 & 25 \\ 2 & 6 & 5 & 4 \\ 3 & 0 & 6 & 1 \end{array}\right] \quad R_1(\tfrac{1}{3}) \to R_1$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 5 & 3 \\ 7 & 12 & 39 & 25 \\ 2 & 6 & 5 & 4 \\ 3 & 0 & 6 & 1 \end{array}\right] \quad \begin{array}{l} R_2 + (-7)R_1 \to R_2 \\ 3R_3 + (-2)R_4 \to R_3 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 5 & 3 \\ 0 & -2 & 4 & 4 \\ 0 & 18 & 3 & 10 \\ 3 & 0 & 6 & 1 \end{array}\right]$$

14. x = 20, y = -11, u = 5, and z = -2

15. $K = \begin{bmatrix} 7 & -8 & 5 \\ -24.5 & 27 & -8.5 \\ 105 & -100 & 19 \end{bmatrix}$

16. There is more than one solution for this problem. As long as matrix A and B are not square matrices and the number of rows in matrix B is equal to the number of columns in matrix A, you will have a solution.

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 3 & 4 & 2 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 2 & 4 & 6 \\ 2 & 8 & 4 & 2 \\ 1 & 4 & 5 & 6 \end{bmatrix}$$

17. D + C = not possible: not same dim.

$$D - 3B = \begin{bmatrix} -2 & 1 & -9 \\ -1 & -3 & -1 \end{bmatrix}$$

$$DC = \begin{bmatrix} 1 & -6 \\ 7 & 6 \end{bmatrix}$$

DA = not possible: the number of rows in A is not equal to the number of cols. in D.

$$B + C^T = \begin{bmatrix} 2 & -1 & 7 \\ -2 & 4 & 0 \end{bmatrix}$$

B^{-1} not possible B is not square.

$$A^{-1} = \begin{bmatrix} 1 & 0 \\ -.5 & -.5 \end{bmatrix}$$

E^{-1} not possible, singular matrix.

18. (a) $WP = \begin{bmatrix} 68.05 \\ 60.10 \end{bmatrix}$

    (b) Each number represents the hourly rate for each crew. John's crew has an hourly rate of \$68.05 and Matt's crew has an hourly rate of \$60.10.

19. (a) x = -14, y = 39, z = -9

    (b) x = -12, y = 37, z = -10