M412 Assignment 6, due Friday October 28

1. [10 pts] Show that for the eigenvalue problem

$$(p(x)u_x)_x + q(x)u + \lambda\sigma(x)u = 0; \quad a \le x \le b,$$

eigenvalues λ are related to their eigenfunctions u by the *Rayleigh quotient*

$$\lambda = \frac{\int_a^b (p(x)u_x^2 - q(x)u^2)dx + (p(a)u(a)u_x(a) - p(b)u(b)u_x(b))}{\int_a^b \sigma(x)u(x)^2 dx}.$$

2. [10 pts] Haberman 2.5.10.

3. [10 pts] Haberman 2.5.14.