M412 Assignment 8, due Friday November 11

1. [10 points] Finish our proof of Fourier's Theorem by showing that

$$\lim_{N\to\infty} \frac{1}{2L}\int_x^{x+L} f(y)\Big(1+2\sum_{n=1}^{N}\cos(\frac{n\pi}{L}(y-x))\Big)dy = \frac{1}{2}f(x^+).$$

2. [5 points] Finish our proof regarding term-by-term integration of the full Fourier series by computing b_n.

3. [5 points] Using Fourier's Theorem, prove that the Fourier sine series for a piecewise smooth function $f(x)$ defined on $[0,L]$ converges to $f(x)$ on $(0,L)$. Under what condition on $f(x)$ does the Fourier sine series definitely not converge at the endpoints $x=0$ and $x=L$?

4. [10 points] For the heat equation

$$\begin{aligned} u_t &= u_{xx} \\ u(t,0) &= 0 \\ u(t,L) &= 0 \\ u(0,x) &= f(x), \end{aligned}$$

where $f(x)$ is assumed continuous on $[0,L]$, with $f(0)=f(L)=0$, and $f'(x)$ is assumed piecewise continuous on $[0,L]$, prove that the infinite series found by the method of separation of variables is a solution. You may use without proving it that under these conditions on f the Fourier sine series associated with f is uniformly convergent.

5. [10 points] Haberman 3.4.4.

6a. [10 points] Haberman 3.4.11.

6b. [5 points] For the PDE in Haberman 3.4.11, find the equilibrium solution $\bar{u}(x)$ and show that it matches the limit of your full solution as $t\to\infty$.

7a. [10 points] Haberman 3.4.12.

7b. [5 points] For the PDE in Haberman 3.4.12, find the equilibrium solution $\bar{u}(x)$ and show that it matches the limit of your full solution as $t\to\infty$.