The Longest Parachute Jump

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(Partial) Example Report

1 Introduction

The longest parachute jump ever made was performed by U.S. Air Force Colonel Joseph W. Kittinger, Jr., on August 16, 1960, in the New Mexico desert. Colonel Kittinger jumped 102,800 feet from an open balloon gandola, and landed successfully after a fall lasting 13 minutes and 45 seconds. In this project, we use Newtonian mechanics to describe the dynamics of Colonel Kittinger's jump.

We take as our starting point Newton's Second Law of Motion, and we consider two forces: gravity and air resistance. For the force due to gravity, we use Newton's Law of Gravitation, observing at the outset that for the scope of Kittinger's jump we must incorporate change in gravitational force as height above the earth varies. For the force due to air resistance, we use dimensional analysis to derive an appropriate model, and we find that the dynamics associated with this term are strongly influenced by air density. For the scope of Kittinger's jump, air density will vary significantly with height above sea level, and in order to capture this effect we develop a model of air density as a function of height. Having arisen from dimensional analysis, our force due to air resistance naturally includes a constant to be determined from data. We assume this constant is different for each of Kittinger's two parachutes (stabilization chute and main chute; we do not separately account for his pilot chute), and we use data from reference [1] and nonlinear regression to find values for these two constants. As depicted in Figure 1, our final model effectively describes the full dynamics of Colonel Kittinger's jump, from step-off to touch-down. Using our model, we are able to determine several dynamical features of Colonel Kittinger's jump, such as the time at which he achieved maximum velocity, the maximum velocity achieved, and the velocity with which Colonel Kittinger touched down.

2 Analyzing the Data

For this project, our data is taken from the *National Geographic* article "The Long, Lonely Leap," listed as reference [1]. Our first data point is the initial height of 102,800 feet, which corresponds with about 31,333 meters.[1] If we let $y(t)$ denote height above the ground, then we have

$$y(0) = 31,333 \text{ m},$$

which will serve as our initial condition for the time evolution.

Our second prospective data point is problematic. On p. 854 of the article (in a figure description at the bottom of the page), Kittinger states that his stabilization chute was on a timer that opened it after 16 seconds. Subsequently, on p. 856 (again in a figure description at the bottom of the page), Kittinger states that the chute opened at a height of 96,000 feet, or about 29,261 meters. This suggests a second data point

$$y(16) = 29,261 \text{ m}.$$

This corresponds with a drop of 2,072 meters in 16 seconds, and we can compare this with the free-fall drop of an object near the earth's surface, which would be

$$\frac{1}{2} g 16^2 = 1,256 \text{ m}.$$

We see that this data point is simply not possible, and we modify it as follows. We assume the stabilization chute did indeed open after 16 seconds, but that the height at which it opened was reported incorrectly. In order to estimate the correct height, we assume negligible air resistance for the first 16 seconds, and solve

$$\begin{aligned} y'' &= -\frac{GM}{(R+y)^2} \\ y(0) &= 31,333 \\ y'(0) &= 0, \end{aligned}$$

from which we compute $y(16) = 30,088$. Here, we use the standard values

$$\begin{aligned} R &= 6.371 \times 10^6 \text{ m (Earth's approximate radius)} \\ M &= 5.9722 \times 10^{24} \text{ kg (Earth's approximate mass)} \\ G &= 6.67408 \times 10^{-11} \text{ m}^3\text{kg}^{-1}\text{s}^{-2} \text{ (approximate value of Newton's gravitational constant).} \end{aligned}$$

The MATLAB M-file used for this calculation is *kitt1.m*, included below.

```
%KITT1: MATLAB Script M-file at the start of
%the Kittinger project, used to identify the
%data point at t=16.
G = 6.67408e-11; M = 5.9722e24; R = 6.371e6;
yprime = @(t,y) [y(2);-G*M/(R+y(1)).^2];
[t y] = ode45(yprime,[0 16],[31333 0]);
y(end,1)
```

[1] In all cases, our conversion factor for converting feet to meters is .3048.

We found two additional data points in the article, the first associated with the time at which Colonel Kittinger's main chute opened, and the second associated with his touchdown. In particular, in the first column on p. 869 of the article, Colonel Kittinger states that his main chute opened after 278 seconds, at an altitude of 5,486 meters. Likewise, he states on the first page of the article (p. 854) that he touched down after 825 seconds. We summarize the data in Table 1.

Time (seconds)	Height (meters)
0	31,333
16	30,088
278	5,486
825	0

Table 1: Data for Kittinger's Jump

3 Developing the Model

Etc., except that I want to give an example of how to incorporate a figure. See Figure 1.

References

[1] J. W. Kittinger, The Long, Lonely Leap, *National Geographic* **118** (1960) 854-887.

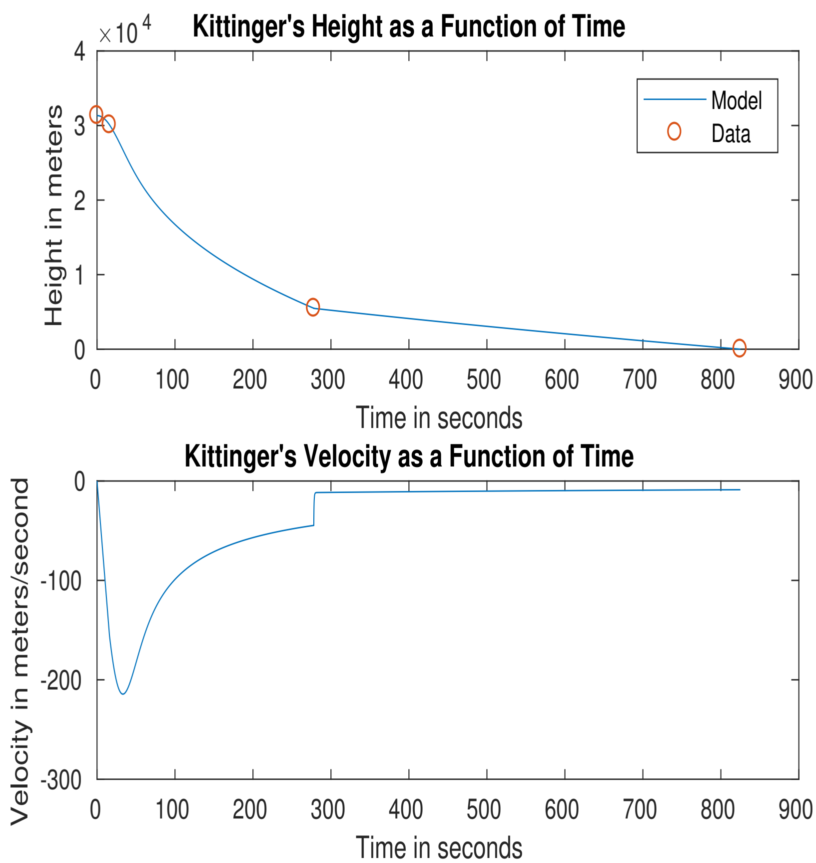

Figure 1: Dynamics for Kittinger's full jump.