Marble Descent through a Viscous Fluid

M442, Spring 2022

Due Friday, March 4

Overview

In this project, we consider an experiment in which marbles are dropped through a viscous fluid and the terminal velocities of the marbles are recorded. Our goal will be to use dimensional analysis and regression to determine a model for these terminal velocities as a function of the given parameters. (**Note:** While it's possible[1] using Stokes' Law, Archimedes' Principle, and Newton's second law of motion to model this phenomenon with ordinary differential equations, that is not the goal of this project.)

Since the viscosity of a fluid is critical to this project, I'll discuss it briefly here. Roughly speaking, viscosity measures the resistance to motion due to collisions between particles. Imagine yourself as a particle walking across campus: density is a measure of how many people are walking around, and viscosity is a measure of how many times they're stopping to talk. In this way a campus crowded with people who do not know one another would have a high density and a low viscosity, while a campus with a few people who know each other well would have a low density and a high viscosity.[2]

In order to understand the more precise definition of viscosity, consider an infinitesimally thin plate with area A moving with velocity v through a viscous fluid, parallel to a flat surface, and separated from the flat surface by a distance h. (Something like a frisbee over a soccer field, the frisbee a height h from the ground.) Viscosity creates a drag between the fixed surface and the thin plate, and in order to keep the plate moving with velocity v, a force F must be applied. We find by experiment that for a certain class of fluids (*Newtonian fluids*, including all gases and many common liquids such as water and light motor oil), this force is proportional to the quantity $\frac{Av}{h}$; i.e., there exists a constant μ so that $F = \mu A \frac{v}{h}$. The value of the proportionality constant μ is generally taken as a measure of fluid viscosity. (For non-Newtonian fluids (e.g., mud, milk, blood, oobleck) the viscosity changes with changing force, and consequently we can't talk about *the* viscosity of the fluid.)

The common metric unit of viscosity is the *poise*, which is defined as the force in dynes[3] required to move a surface that is one square centimeter in area past a parallel surface at a speed of one centimeter per second, with the surfaces separated by a fluid film one centimeter thick (i.e., the units are dyne $\cdot$ s/cm^2, with associated dimensions $ML^{-1}T^{-1}$). The SI unit

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[1]In fact, pretty easy.

[2]Unless the people don't like each other, but you see what I'm getting at.

[3]One dyne is the amount of force required to give a free mass of one gram an acceleration of one centimeter per second per second.

for viscosity is actually pascal-seconds, but this is seldom used in practice. Of course, all this is for a fixed temperature, and viscosity changes dramatically with temperature.

Experimental Data

Data was collected for two fluids and five marbles. Physical properties of the fluids and marbles are listed in Table 1, while terminal velocities are tabulated in Table 2.

- **Marble 1.** Mass $m = .003$ kg, radius $r = .0070$ m.

- **Marble 2.** Mass $m = .005$ kg, radius $r = .0080$ m.

- **Marble 3.** Mass $m = .013$ kg, radius $r = .0110$ m.

- **Marble 4.** Mass $m = .022$ kg, radius $r = .0130$ m.

- **Marble 5.** Mass $m = .054$ kg, radius $r = .0175$ m.

- **Fluid 1.** Kroger Light Corn syrup, density $\rho = 1427.1$ kg/m^3, viscosity $\mu = 5.0$ kg/m/s.

- **Fluid 2.** Suave Daily Clarifying Shampoo, density $\rho = 1018.0$ kg/m^3, viscosity $\mu = 20.0$ kg/m/s.

Table 1: Physical properties of fluids and marbles.

Fluid/Marble	Marble 1	Marble 2	Marble 3	Marble 4	Marble 5
Corn syrup	.026 m/s	.030 m/s	.046 m/s	.070 m/s	.076 m/s
Shampoo	.009 m/s	.010 m/s	.016 m/s	.019 m/s	.021 m/s

Table 2: Experimental terminal velocities.

Assignments

1. Use dimensional analysis to determine a general form for the terminal velocity of a marble descending through a viscous fluid. (The first step here consists of deciding what the key variables are; of course in this regard the data I've given you is a pretty good hint.) Your report should contain a typeset version of this analysis.

2. Use the data in Tables 1 and 2 to develop a predictive model of the terminal velocity for any marble and fluid. Your report should contain a tabulation of dimensionless products used for this calculation and any plots you used in the fitting. (For some approaches to this problem plots aren't particularly enlightening, and if you take such an approach you don't need to submit any plots.)

3. Plot your model terminal velocities along with the data terminal velocities. Take as your horizontal axis experiment numbers 1, 2, ..., where the first experiment is corn syrup with marble 1, the second is corn syrup with marble 2, ..., the sixth is shampoo with marble 1 etc.

4. Compute the SSR for your fit

$$E = \sum_{k=1}^{10} (v_k - \hat{v}_k)^2,$$

where the values $\{v_k\}_{k=1}^{10}$ denote the ten experimentally obtained terminal velocities (from Table 2), and the values $\{\hat{v}_k\}_{k=1}^{10}$ denote the ten corresponding values according to your model. In addition, compute the standard deviation

$$s = \sqrt{\frac{E}{10 - m}},$$

where m denotes the number of parameters in your model. You should expect to get a value for s smaller than .02.

5. Write a MATLAB function M-file that takes as input the appropriate variables, as determined in Item 1 above, and returns the terminal velocity associated with a particular marble moving through a particular fluid. In class on Friday, Mar. 4, you will test your model by predicting the terminal velocity for the marble and fluid presented in class. You will write your prediction at the top of your report, and your project grade will be based entirely on its accuracy.[4]

Note on Project Reports

Although key steps associated with this project were enumerated in the previous section of this assignment (for clarity), your project report should not have numbered responses. Rather, your report should be written in the spirit of a journal publication with the following sections:

- An Introduction or Overview that provides a summary of both the project and your results.

- One or more (typically more) section(s) of analysis in which you describe how you proceeded. For example, it would be natural in this project to carry out your dimensional analysis in one section and your associated regression fit in another.

- A final Summary section in which you interpret your results. The primary difference between the summary at the end and the overview at the beginning is that for the summary you are allowed to assume the reader is familiar with all of your calculations. For the overview, you need to assume you're addressing someone who has no idea what

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[4]Just kidding.

your data looks like, what your dimensionless products are, etc. So, for example, it would be appropriate to discuss alternative dimensionless products in your summary, but not in your overview.

In addition to this basic layout, you should keep the following points in mind while writing your reports:

- A partial example report (for a project that won't be assigned this semester) is available on the course web site.

- Scientific articles are typically written in first person plural, present tense. For example, here's a short passage from an article by René Descartes (French philosopher and mathematician, 1596-1650) entitled *Rules for guiding one's intelligence in searching for the truth*: "...we will never become mathematicians, even if we remember all the demonstrations of others, unless we are also able to use our intelligence to solve whatever problems we encounter." The idea behind this is that you should view yourself as taking the reader on a tour of your work. So, if you were giving a tour of Texas A&M University, you might start by saying something like, "We're starting at the top of Rudder Tower to get an overview of campus. From here, we can see..."