M469 Spring 2020, Assignment 2, due Fri., Jan. 31

Suggested Reading. *Uses and abuses of mathematics in biology*, by R. M. May, in *Science* **303** (2004) 790-793. First, the author of this article, Robert May, has done a good bit of important work in mathematical biology. We'll read two of his technical papers in the next two weeks. In this non-technical article he discusses the role mathematics can play in clarifying our thoughts about biological processes, but he also warns that the way in which mathematics is often currently used can lead to erroneous conclusions. Though he discusses the Navier-Stokes equations in the context of meteorology, keep in mind that they also arise in the study of blood flow.

1. Answer the following.

a. [5 pts] Show that for any value $a \neq 1$ and positive integer t

$$\sum_{i=0}^{t-1} a^i = \frac{a^t - 1}{a - 1},$$

and show that this relation remains valid in the limit as $a \to 1$.

b. [5 pts] Find an explicit solution formula for the first order linear difference equation

$$y_{t+1} = ay_t + b.$$

2. [10 pts] Granted, there is no biological content to this problem, but this is certainly the most common situation in which the solution from Problem 1 arises. An amortized loan is one in which an initial principal P_0 is borrowed at some interest rate r, and payments with value v are made at equal time intervals, often monthly. (For example, car and house payments typically work this way.)

a. Write down a difference equation that models this scenario and write down its solution. Assume r denotes interest rate for the payment period, so for example if the annual interest rate is .05, as in Part (b) below, and the payments are monthly, then $r = .05/12$. (To be clear, it's generally assumed in these situations that when we say annual rate r_A we mean monthly rate $\frac{r_A}{12}$, even though monthly growth at rate $\frac{r_A}{12}$ isn't quite the same as annual growth at rate r_A.)

b. If a car loan is $P_0 = 20,000$ for five years at annual rate .05, what must the monthly payment v be?

3. [10 pts] Ricker's single-species population model is

$$y_{t+1} = y_t e^{r(1-\frac{y_t}{K})}.$$

a. Transform this equation into a form for which linear regression could be applied in estimating the parameter values r and K. Explain, in principle, how you would refine your estimate with nonlinear regression.

Note. For this discussion you can assume your data has the form $\{(t_k, \bar{y}_k)\}_{k=1}^N$ and, as opposed to our U.S. population data from class, that you have a data point for each unit of time. I.e., $t_1 = 0$, $t_2 = 1$, etc.

b. If Ricker's model is used to model U.S. population data we find, from the nonlinear fit,

$$\begin{aligned} r &= .0208 \\ K &= 484.9767 \\ y_0 &= 8.2399. \end{aligned}$$

Compare this result with the 2010 birth, death, and immigration/emigration rates we discussed in class for the U.S. population. By the way, Ricker's model predicts a population of 327.0182 for 2020.

4. [10 pts] Consider the Hassell-type recursion equation

$$y_{t+1} = \frac{ay_t}{(1 + by_t)^2},$$

where $a > 1$, $b > 0$. Does this model have a carrying capacity, and if so, what is it?