M469 Spring 2020 Assignment 3, due Fri. Feb. 7

Suggested Reading 1. *Population growth and Earth's carrying capacity*, by Joel E. Cohen, in *Science* **269** (1995) 341-346. This article discusses various aspects of world population growth, including physical considerations, predictions of carrying capacity, and a model in which carrying capacity changes with time. Note particularly the suggested range for Earth's carrying capacity: 7.7 billion to 12 billion. Also, check out endnote 16 for a mention of how some authors went wildly off the rails by using the logistic model to fit global population growth.

Suggested Reading 2. *Biological populations with nonoverlapping generations: stable points, stable cycles, and chaos*, by Robert M. May, in *Science* **186** (1974) 645-647. This is the first of two technical articles we'll read by May about single difference equations. To some extent our section on single difference equations in class will consist of clarifying May's discussion. Note particularly that his Equation (1) is the model now generally attributed to Ricker, and his Equation (2) is Verhulst's logistic model.

1. [10 pts] Non-dimensionalize Ricker's model

$$y_{t+1} = y_t e^{r(1-\frac{y_t}{K})},$$

and draw cobwebbing diagrams for your nondimensional form for two different choices of r, one for which solutions oscillate and one for which solutions do not oscillate. In both cases take $Y_0 = \frac{1}{2r}$, where Y_t denotes your dimensionless variable.

2. [10 pts] Suppose a certain population in the presence of predation can be modeled by

$$y_{t+1} = \frac{ry_t^2}{M + y_t^2},$$

where $r > 0$ and $M > 0$. Nondimensionalize this equation in such a way that only one (combined) parameter remains. For what values of your parameter is extinction guaranteed for all initial populations? For values of your parameter for which extinction is not guaranteed, find the threshold population under which extinction is guaranteed.

3. [10 pts] In class we non-dimensionalized the discrete logistic model

$$y_{t+1} = y_t + ry_t(1 - \frac{y_t}{K})$$

as

$$Y_{t+1} = Y_t + rY_t(1 - Y_t)$$

by setting $Y_t = \frac{y_t}{K}$. We can obtain a more compact form if we first re-write the discrete logistic model as

$$y_{t+1} = (1 + r)y_t(1 - \frac{r}{(1 + r)K}y_t).$$

Carry out the natural non-dimensionalization for this last equation, and set $R = 1 + r$ in the form you obtain. Working directly with your non-dimensionalized equation, find all fixed points and determine values of R for which each is stable. Discuss what your analysis suggests about possible bifurcations, and check that your result agrees with our calculations from class.

4. [10 pts] **The Allee Effect.** In some cases if the number of individuals in a species falls below a certain threshold, members of the species will have trouble finding suitable mates, and the species will die out. This is known as the Allee effect, named after the U.S. ecologist Warder Clyde Allee (1885-1955). If we let L denote this threshold, then a reasonable single-species discrete model incorportating the Allee effect is

$$y_{t+1} - y_t = r y_t \left(1 - \frac{y_t}{K}\right)\left(\frac{y_t}{L} - 1\right).$$

Non-dimensionalize this model, and show that you can write your non-dimensionalized equation in the form

$$Y_{t+1} - Y_t = a Y_t (1 - Y_t)(Y_t - b),$$

for appropriate choices of a and b. Explain why you should have $0 < b < 1$. For each of the fixed points for this last equation, determine values for a and b for which it is stable. Discuss what your analysis suggests about bifurcations.