M469 Spring 2020 Assignment 8, due Fri. Apr. 3

Suggested Reading. *Experimentally induced transitions in the dynamic behavior of insect populations*, by R. F. Costantino et al, in *Nature* **375** (1995) 227-230. The authors consider a nonlinear age-structured population model (difference equation) for the flour beetle Tribolium. The model has six parameters, which the authors initially estimated from data. Next, the authors selected one parameter—μ_a, the probability of an adult beetle dying from a cause other than cannibalism—and manipulated it in the physical population by removing individual beetles (hence increasing the active death rate). In this way, they could carry out a bifurcation analysis with both their model and the observed population, and they could determine whether or not the predicted bifurcations were actually physical. In their Figures 2 and 3 they show that three types of behavior correspond (correspondence between the model's prediction and the population's behavior): stable fixed points, stable 2-cycles, and chaotic behavior (which they describe as aperiodic oscillations).

1. [10 pts] Use your result from Problem 4 in Assignment 7 to show that the Kimura 2-parameter distance is

$$d_{K2}(S_0, S_t) = -\frac{1}{4}\ln(1-2p_2) - \frac{1}{2}\ln(1-2p_1-p_2),$$

where p_1 denotes the probability of a transition (at a particular site) in t time steps and p_2 denotes the probability of a transversion (at a particular site) in t steps. Using this distance, compute the Kimura 2-parameters distance between the two strands of DNA given in Problem 1 of Assignment 7.

2. [10 pts] Suppose we have four taxa S_1, S_2, S_3, and S_4, and we have compute the Jukes-Cantor distances between every pair:

	S_2	S_3	S_4
S_1	1.2	.9	1.7
S_2		1.1	1.9
S_3			1.6

Use the UPGMA method (i.e., the method we discussed in class) to suggest an appropriate phylogenetic tree relating these taxa.

3. [10 pts] The discrete Lotka-Volterra competition model is

$$\begin{aligned} y_{1_{t+1}} - y_{1_t} &= r_1 y_{1_t}\left(1 - \frac{y_{1_t} + s_1 y_{2_t}}{K_1}\right) \\ y_{2_{t+1}} - y_{2_t} &= r_2 y_{2_t}\left(1 - \frac{s_2 y_{1_t} + y_{2_t}}{K_2}\right). \end{aligned}$$

a. Transform this equation into a form for which linear regression could be applied in estimating the parameter values. Explain, in principle, how you would refine your estimate with nonlinear regression. Denote your data $\{(t_k, \vec{p}_k)\}_{k=1}^{N}$.

b. Non-dimensionalize this equation.

4. [10 pts] The discrete Lotka-Volterra predator-prey model is

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\begin{aligned}
y_{1_{t+1}} - y_{1_t} &= a y_{1_t} - b y_{1_t} y_{2_t} \\
y_{2_{t+1}} - y_{2_t} &= -\, r y_{2_t} + c y_{1_t} y_{2_t},
\end{aligned}
$$

where the parameter values are all positive. Non-dimensionalize this equation and then find the fixed points for your non-dimensional form. Find the range of parameters for which each fixed point is stable and the range for which each is unstable. Check your work with our analysis from class by taking a limit as $K \to \infty$ in our model from class.