ABSTRACT & SUBJECT CLASSIFICATION

APPLICANT NAME	DATE SUBMITTED
Sottile, Frank	1/17/2019 10:31:47 PM

TITLE OF PROJECT *(Titles exceeding 81 characters, including spaces and punctuation, will be truncated.)*

Applicable and Combinatorial Algebraic Geometry

The current research of Sottile involves over two dozen collaborators, most are not at Texas A&M and on many are abroad. They work on projects in the applications of algebraic geometry and combinatorial aspects of algebraic geometry. These themes are linked. Many common algebraic varieties, including those arising in applications, have aspects of their structure strongly controlled by combinatorics. A deeper understanding of such varieties has implications beyond that study. It leads to new combinatorics and reveals hidden aspects of the geometry. These combinatorial structures form the backbone of symbolic and numerical algorithms to represent and manipulate such combinatorial algebraic varieties. This deep understanding of such varieties and the use of algorithms in turn provide insight to address questions from the rest of mathematics and the applied sciences.

Sottile's work in the applications of algebraic geometry includes the development of algorithms in numerical algebraic geometry. With Burr of Clemson and student Walker, he is using the theory of Newton-Okounkov bodies to design algorithms for solving systems of polynomial equations, based on degenerations into toric varieties. With Leykin of Georgia Tech, Hauenstein of Notre Dame, and Rodriguez of Wisconsin, Sottile is working to develop and use generalized witness sets for manipulating subvarieties of homogeneous spaces, such as multiprojective varieties. With Rodriguez and student Brysiewicz, he is developing a polyhedral homotopy algorithm that exploits the Galois group of a sparse system. Another area of applications is to develop splines in the new directions of semialgebraic splines and quasipolynomial splines with DiPasquale of Colorado State.

Sottile's work in combinatorial aspects of algebraic geometry is in several directions. With Bridy of Yale and student Yahl, he is using geometry, number theory, and computation to understand and determine Galois groups of sparse polynomial systems and those in Schubert calculus. This includes a possible classification of Schubert problems and sparse systems by Galois group, along with solving the inverse Galois problem in these contexts. Developing combinatorics related to Schubert Galois groups is a project with student Ying and Martin del Campo of CIMAT. More traditional combinatorial aspects of Schubert calculus involve collaborators Colmanarejo of Massachusetts and Bergeron of York University in Toronto towards Murnaghan-Nakayama formulas in quantum cohomology and in K theory of the flag manifold. With Li of Guangzhou and Ravikumar of Chennai, he is working on Pieri formulas in equivariant cohomology in all Lie types. A continuing study of the structure of amoebas and coamoebas from tropical geometry is in collaboration with Nisse of Xiamen University in Malaysia.

Recent Work of Frank Sottile

In the past five years, I have completed or published 29 research and two expository articles. This has been with 32 different coauthors. I discuss six papers/themes in this work.

Foundations of numerical algebraic geometry. Numerical algebraic geometry uses methods from numerical analysis to represent and manipulate algebraic varieties on a computer [29]. It holds great promise as a tool for the applications of algebraic geometry in part because its algorithms are easily parallelized. I have been interested in foundational aspects of this subject, both to better understand its basic notions and to develop new algorithms. One of its fundamental algorithms is to numerically compute an irreducible decomposition of a variety, using a combination of monodromy and computing certain linear traces. Leykin, Rodriguez, and I extended this algorithm to subvarieties of products of projective spaces, using an version of witness sets given by Hauenstein and Rodriguez [11]. We found a simple derivation of the trace test and a significant improvement of the decomposition algorithm [21] when there are two factors. We four are now working to extend this to many factors.

I have also sought to design and implement new algorithms for solving that exploit theoretical advances, such as the Gale transform in fewnomial theory [3] or Vakil's geometric Littlewood-Richardson rule [20]. With Hauenstein and Rodriguez, we have developed and implemented an algorithm to compute a generating set for the monodromy group of a branched cover [12], which relies on a theorem of Zariski.

Certification in numerical algebraic geometry. Computations in numerical algebraic geometry rely upon floating-point arithmetic and approximations to solutions of systems of equations (points on a variety). The inexact nature of the computation is one of its strengths, but a consequence is that the computations are not *a priori* certified to be correct. Smale [28] showed how to certify the solutions to systems of polynomials with the same numbers of equations as variables (square systems), and this has been implemented [13]. I have been interested in extending this to more general overdetermined systems. Hauenstein, Hein, and I showed how to use Grassmann duality to certify problems in the Schubert calculus [10]. Hein and I found a different scheme for Schubert calculus [15], which should also work for any degeneracy locus, as it parameterizes the kernel of a matrix, lifting to a square system. Recently, Duff and I showed how to use some knowledge about a Newton-Okounkov body [19] to certify an overdetermined system [7].

Applications of algebraic geometry. I have other work in applications. A continuing interest is to better understand real solutions to systems of equations and geometric problems. One of the most curious phenomena that I discovered in computational experiments were congruences and other restrictions satisfied by the numbers of real solutions to some systems of equations. With Hein and Zelenko, we found a geometric reason [16, 17] for congruences modulo four in Schubert calculus observed in experimentation [14].

I helped Kuchment's student Do with a very challenging numerical computation for her thesis—it showed that a certain conjectural nondegeneracy of the spectrum for generic periodic Schrödinger/Laplace-Beltrami operators held for a particular discretization. This conjecture, while widely believed, had not been demonstrated to hold in any meaningful example. While refining the computation and trying to justify the argument, we realized that some insight from real algebraic geometry together with a property of many discretizations, proves this nondegeneracy conjecture for many discretizations [6].

Schubert calculus. The Schubert calculus of enumerative geometry is a well-studied class of geometric problems with important combinatorial properties. I continue to be interested in combinatorial formulas and positivity in Schubert Calculus. Adeyemo, a mathematician I met in Nigeria, worked with me to show how the geometric pattern map of Billey and Braden [4] gives formulas in cohomology, K-theory, and their equivariant cousins [1, 2]. Previous work with Bergeron for cohomology and Lenart for K-theory shows that these formulas, when combined with Pieri-type formulas in type A, give new formulas for Schubert classes. The relevant Pieri-type formulas are not yet known in general Lie type. I am working with Li in Guangzhou and Ravikumar in Chennai on such Pieri-type formulas in equivariant cohomology for general Lie type; we have completed this in type A [22].

Galois groups in enumerative geometry. Camille Jordan explained how problems in enumerative geometry have Galois groups [18], and observed that the structure of the Galois group encodes subtle structure in the solutions. These algebraic Galois groups coincide with geometric monodromy groups. A decade ago, Leykin and I used numerical algebraic geometry to compute monodromy for some Schubert problems on Grassmannians, and I have been studying them since. As with univariate polynomials, most Schubert problems have full symmetric Galois groups, and only a few have smaller Galois groups; we call these problems *enriched*. With Martín del Campo and Brooks, we showed that all Schubert problems on $G(2, n)$ have Galois groups containing the alternating group [5], and therefore are not enriched. White and I were able to show that many Schubert problems, including all on $G(3, n)$, have Galois group that acts doubly transitively on the solutions [30], and thus are likely not enriched. With Martín del Campo and Williams, we classified and identified the Galois groups of all enriched Schubert problems on $G(4, 8)$ and $G(4, 9)$—only 154 of the $81{,}000^+$ problems on the latter space are enriched [23]. This suggests the possibility of classifying all enriched Schubert problems, as well as solving the inverse Galois problem for Schubert calculus.

A consequence of this is a dichotomy—either a Schubert Galois group is the full symmetric group or it acts imprimitively. Esterov [8] proved this dichotomy for systems of sparse polynomial equations. The solutions to such systems may be understood using techniques from toric varieties, but it is not known in general what are their Galois groups. A motivation for my work with Hauenstein, Rodriguez on numerical computation of Galois gruops [12] was to develop topols for this study.

Combinatorial algebraic geometry. Most of my work involves algebraic varieties with a strong combinatorial structure, studying and exploiting those structures for theoretical purposes or applications. I describe work related to tropical geometry. Writing a non-zero complex number as $e^{r+i\theta}$ identifies $\mathbb{C}^\times$ with $\mathbb{R} \times S^1$. The projection of a variety $V \subset (\mathbb{C}^\times)^n$ to its lengths in $\mathbb{R}^n$ is its amoeba $\mathcal{A}$. The structure of $\mathcal{A}$ at infinity is encoded by the logarithmic limit set or tropical variety of V, which is a rational polyhedral fan. Nisse and I discovered a similar structure, the phase limit set, for the coamoeba of V, which is its projection to $(S^1)^n$, the set of angles in V [24]. We used this to show that the complement of coamoeba has the subtle homological/geometric property of being higher convex [25], as defined by Gromov [9]. We also established the higher convexity of complements of many tropical varieties [26]. Our recent research ran up against some fundamental open questions about the structure of amoebas and coamoebas, which we explain in a paper that also takes initial steps towards addressing these questions [27].

References

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15. N. Hein and F. Sottile, *A lifted square formulation for certifiable Schubert calculus*, J. Symbolic Comput. **79** (2017), no. part 3, 594–608.
16. N. Hein, F. Sottile, and I. Zelenko, *A congruence modulo four in real Schubert calculus*, J. Reine Angew. Math. **714** (2016), 151–174.
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23. A. Martín del Campo, F. Sottile, and R. Williams, *Classification of Schubert Galois groups in $G(4,9)$*, 2019.
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Travel/Invitation Goals
September 2019—August 2020
Frank Sottile

I discuss my major travel plans for the period September 2019 through August 2020.

I will attend the BIRS-CMO workshop on "Tropical Methods in Real Algebraic Geometry" at Casa Matematica Oaxaca from 8 to 13 September 2019. This meeting touches on two of my current and long-standing research interests.

Later the same month, I will attend a Oberwolfach workshop on "Toric Varieties" from 22 to 29 September. As with the CMO meeting, studying and using toric varieties is a significant theme in my current research interests. I do not expect other major travel in the Fall, but may invite collaborators, such as Andrew Bridy, a postdoc at Yale or Laura Colmenarejo, a postdoc at U. Mass.

I will spend January–May 2020 at the MSRI in Berkeley; my wife is an Eisenbud Professor in their program on "Quantum Symmetries" that semester, and I have been invited as a member of the complimentary program. As I have done when there in the past, I expect to visit many of the math circles in the Bay Area. I will also likely travel to meet collaborators, or have some come to Berkeley to work with me.

In March 2020, I may travel to Ouagadougou, Burkina Faso for two weeks for a CIMPA Research School. With others in the Simons Foundation supported PREMA collaboration in West Africa, we have applied to the CIMPA to run a school on symmetric functions and combinatorics on words; the decision both on the school, and on its dates have not yet been announced. As lecturing at a CIMPA school is considered a service to the profession, I am expected to fund my own travel (and the same for whenever I attend other events with PREMA).

In June 2020, I will travel to Vancouver Canada to attend the triennial meeting on Foundations of Computational Mathematics; the meeting is 15–24 June at Simon Fraser University. I may organize a session at that meeting.

I am the organizer of a summer school and research conference on nonlinear algebra at KAIST in South Korea from 12–20 August 2020. This is related to the semester at the ICERM that was just organized in Fall 2018 and will include lecturers and students from North America and Europe, in addition to Korea and East Asia.

On top of these scheduled trips, I am making plans to spend about 2 weeks in Malaysia in 2020 sometime before August. A collaborator and former postdoc, Mounir Nisse, is starting a professorship at Xiamen University Malaysia this semester, and we are planning for me to visit him for research, and also to visit other Universities in Kuala Lumpur, which would also include outreach activities such as public lectures and running activities for high-school students. This would be similar to trips I have made to Nigeria and the Republic of Benin to with engage the mathematical community in those countries.

Curriculum Vitae: Frank Sottile

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Professional Preparation

Michigan State University, Honors B.S. in Physics, 1985.
University of Cambridge, M.S., Maths Tripos Part III, with distinction, 1986.
University of Chicago, S.M. Mathematics, 1989,
Ph.D. in Mathematics, 1994.

Thesis Advisor: William Fulton

Research Interests: Real, computational, and combinatorial algebraic geometry; algebraic combinatorics; combinatorial Hopf algebras, discrete and computational geometry, tropical geometry, toric varieties, applications of algebraic geometry.

Professional Experience

Professor, Texas A&M University, Since September 2006.
Visiting Adjunct Professor in the Department of Combinatorics and Optimization at the University of Waterloo, 2014–2018.
Visitor, Technische Universität München, August 2005–July 2006.
Professuer Invité, Université Paris 6, November 2005.
Associate Professor, Texas A&M University, August 2004–August 2006.
Clay Mathematical Institute Senior Researcher, January–June 2004.
Assistant Professor, University of Massachusetts at Amherst, 2000–2004.
Van Vleck Assistant Professor, University of Wisconsin at Madison, 1999–2000.
MSRI Postdoctoral Fellow, 1996–1997 and Autumn 1998.
Term-Limited Assistant Professor, University of Toronto, 1994–1998.

Major awards

Fellow of the American Mathematical Society, 2012.
Department of Mathematics award for service, December 2009.
Kavli Fellow, US National Academy of Sciences' Japanese-American Frontiers of Science Symposium, 2007.
NSF CAREER fellow, 2002–2007.
National Science Foundation Graduate Fellow. 1986–1989.
Winston Churchill Foundation Graduate Fellow, Churchill College, Cambridge University, 1985–1986.

Major research grants

NSF Individual Research Grant, June 2015–May 2018. DMS-1501370.
NSF Individual Research Grant, August 2010–July 2014. DMS-1001615.
NSF Individual Research Grant, August 2009–July 2013. DMS-0915211.
Texas ARP grant: Algebraic Geometry in Algebraic Statistics and Geometric Modeling, May 2008–May 2010.
NSF Individual Research Grant. September 2007–August 2010. DMS-0701050
NSF CAREER grant, August 2002–July 2007.

NSF Individual Research Grant, June 2000–July 2002. DMS-0070494.
NSERC Individual Research Grant. April 1995–June 1998.

Graduate Students: Jim Ruffo (Ph.D 2007), Weronika Buczynska (Ph.D. 2010), Corey Irving (Ph.D. 2012), Abraham Martin Del Campo-Sanchez (Ph.D. 2012), Nikolas Hein (Ph.D. 2013), Robert Williams (Ph.D. 2017), Ata Pir, Li Ying, Taylor Brysiewicz, and Elise Walker.

Postdoctoral Advisees: Greg Warrington (2000-3), Evgenia Soprunova (2002-2004), Frédéric Bihan (2004), Luis Garcia (2005–2007), Christopher Hillar (2005–2008), Zachariah Teitler (2007–2010), Aaron Lauve (2007–2010), Jon Hauenstein (2010–2012), Mounir Nisse (2010–2013), Jacob White (2012–2015), Timo de Wolff (2014–2017), and Emanuele Ventura (2017–).

Major Service activities

(1) Member of organizing committee, "Nonlinear Algebra", ICERM thematic program, Fall 2018.
(2) Member of BIRS Scientific Advisory Board, since October 2017.
(3) Co-organizer (with Drs. H.P. Adeyemo and Erwan Brugallé), CIMPA Research School on Computational and Combinatorial Algebriac Geometry, University of Ibadan, Nigeria, June 2017.
(4) Corresponding editor, SIAM Journal on Applied Algebra and Geometry, since 2016.
(5) Associate Editor, American Mathematical Monthly since 2014.
(6) Co-organizer, TAMU Math Circle, since 2011.
(7) Co-chair, program committee, SIAM Conference on Applied Algebraic Geometry, 6–9 October 2011, North Carolina State University.
(8) Editorial Board, Canadian Journal of Mathematics and Canadian Mathematical Bulletin, 2011–2015.
(9) Founder and Chair, SIAM Activity group on Algebraic Geometry (July 2009–December 2011).
(10) Member of organizing committee, "Real and Tropical Geometry", Bernoulli Centre, Laussanne Switzerland, January–June 2008.
(11) Lead organizer and lecturer, IMA PI Summer Graduate School on Applicable Algebraic Geometry, July 23–10 August 2007.
(12) Chair of the organizing committee, MSRI semester on "Topological Aspects of Real Algebraic Geometry", January-May 2004.
(13) Editorial Board, SIAM Journal on Discrete Mathematics, 2004–2015.
(14) Refereed over 330 papers for over 120 Journals.
(15) Grant review for NSF, NSA, NSERC (Canada), ISF (Israel), FWF (Austria), FONDEYC (Chile), ANR (France).

Publications:

Over 112 publications and one book:
`www.math.tamu.edu/~sottile/research/publications.html`.

Talks:

Over 450 research and colloquium talks: `www.math.tamu.edu/~sottile/talks/talks.html`.
Over 150 outreach presentations: `www.math.tamu.edu/~sottile/talks/outreach.html`.

Applicable and Combinatorial Algebraic Geometry:
Frank Sottile
Five Recent Publications

(1) Anton Leykin, Jose Israel Rodriguez, and Frank Sottile, *Trace test*, Arnold Math. J. **4** (2018), no. 1, 113–125.

(2) Jonathan D. Hauenstein, Jose Israel Rodriguez, and Frank Sottile, *Numerical computation of Galois groups*, Found. Comput. Math. **18** (2018), no. 4, 867–890.

(3) Jonathan D. Hauenstein, Nickolas Hein, and Frank Sottile, *A primal-dual formulation for certifiable computations in Schubert calculus*, Found. Comput. Math. **16** (2016), no. 4, 941–963.

(4) Nickolas Hein, Frank Sottile, and Igor Zelenko, *A congruence modulo four in real Schubert calculus*, J. Reine Angew. Math. **714** (2016), 151–174.

(5) Frank Sottile and Jacob White, *Double transitivity of Galois groups in Schubert calculus of Grassmannians*, Algebr. Geom. **2** (2015), no. 4, 422–445.

Colloquiua, Research Seminars, and Talks at Scientific meetings since 2013.

Since 2013, I have given a total of 107 research talks at colloquia, seminars, and conferences, and 90 outreach talks and public lectures to math circles and math clubs. I list most of the research talks and omit the others.

One-hour invited talks:

- Contemporary Mathematics, Moscow, December 2017
- Texas Algebraic Geometry Symposium, Rice, April 2017.
- CARGO 15th Anniversary, Wilfrid Laurier, March 2017.
- Shanks Workshop on Real Algebraic Geometry, Vanderbilt University, March 2017.
- Mirror Symmetry and Tropical Geometry KIAS, Seoul, Korea, February 2017.
- Convexity in Algebraic Geometry, Fields Institute, October 2016.
- COALGA, Buenos Aires, August 2016.
- AAS/AMU Symposium, Abuja, Nigeria, May 2016.
- Workshop on the Global Attractor Conjecture, San Jose State U., March 2016.
- Fields Institute Workshop, December 2015.
- AMS Summer Institute in Algebraic Geometry, Salt Lake City, July 2015.
- MEGA in Trento, June 2015.
- Joint Mathematics Meetings, January 2015,
- Perspectives of Modern Complex Analysis, Banach Center, Poland, July 2014.
- Nigerian Women's Mathematics Conference, U Ibadan, Nigeria, July 2014.
- Optimization and Algebraic Geometry, NIMS, Daejeon, Korea, June 2014.
- Workshop on Algebraic Geometry, KAIST, Daejeon, Korea, June 2014.
- Applied Geometry, Topology, and Networks, University of Illinois, February 2014.
- Combinatorial Methods in Topology and Algebra, Cortona, Italy. September 2013.
- AlGeCom, IUPUI, Indianapolis, April 2013.
- CAAC-13, Fields Institute, Toronto, January 2013.

Short invited talks: Special sessions and minisymposia: SIAM AG17, 2017 Joint Mathematics meetings, AMS Fall 2016 Eastern Section, SIAM 2016 annual meeting, Approximation theory, AMS Spring 2016 South Eastern Section, AMS Spring 2015 Eastern Section, SIAM AG15, AMS Spring 2014 South Eastern Section, SIAM AG13, 2013 Joint Mathematics meetings.

Colloquia: UT Dallas, Michigan State, Western Ontario, Waterloo, McMaster, Toronto, Santa Clara, IMSP Benin, FUNAAB Nigeria, Ibadan Nigeria, Yale, Michigan, San Jose State, Kansas, U Virginia, U Ibadan, USC, Sam Houston State U, U Pennsylvannia, U Wisconsin, UC Berkeley, Santa Clara U, MSRI.

Short courses:

- CIMPA School at Ibadan, Nigeria, 11–24 June 2017.
- University of Ilorin, Nigeria, 14–16 July 2014.
- Pingree Park Campus, Colorado State University, 29–31 July 2013.

Seminar talks: Simon Fraser, UBC, UT Dallas, Texas A&M, York University, Frankfurt, Fields Institute, Toronto, UBC, Fields Institute, UT Arlington, Texas A&M, Texas A&M, Texas A&M, Texas A&M, Texas A&M, Dalian University of Technology, UT Arlington, Texas A&M, Texas A&M, NYU, UC Berkeley, UC Berkeley, UC Davis, Texas A&M, U Waterloo, NIMS Korea, Texas A&M, U Illinois, Texas A&M, U Pennsylvannia, U Texas at Arlington, U Wisconsin, U Chicago, Texas A&M, Stanford, SFSU, UC Davis, UC Berkeley, UC Berkeley, UC Berkeley.

Recent Ph.D. Students Supervised: Frank Sottile

- Dr. Robert Williams. Dissertation: "Restrictions on Galois Groups in Schubert Calculus", Ph.D. 2017.
  Visiting Assistant Professor, Rose-Hulman Institute of Technology, Terre Haute, Indiana.
- Ata Pir. Dissertation: "Irrational Toric Varieties", Ph.D. August 2018.
  AlphaStar Math Academy, 4701 Patrick Henry Dr #25, Santa Clara, Califotnia.
- Li Ying. (Current) Dissertation: "Stability of Heisenberg coefficients". This is in Combinatorial Representation Theory, Ph.D. expected in 2019.
- Taylor Brysiewicz (Current) working in Applied and Numerical Algebraic Geometry, Ph.D. expected in 2020.
- Elise Walker (Current) working in Applied and Numerical Algebraic Geometry, Ph.D. expected in 2022.

Recent Postdocs Supervised: Frank Sottile

- Jacob White (2012–15). White is an Assistant Professor at The University of Texas Rio Grande Valley.
- Timo de Wolff (2014–2017). De Wolff holds an Emmy Noether Fellowship at the Technische Universität Berlin. He is one of two mathematicians to win this competetive award given by the DFG, which supports his research group for five years. He will assume a professorship at Universität Braunschweig in May 2019.
- Emanuele Ventura (2017–) Ventura is currently a postdoc at Texas A&M.