Two Kinds of Stress Tensor Anomaly

QVac, May 2013

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Classifications of Anomalies

Celestial mechanics

1. The mean anomaly
2. The true anomaly
3. The eccentric anomaly

Bulk quantum field theory

	Gauge	*Gravity*
harmless	axial (Abelian)	conformal
fatal	chiral (non-A.)	Lorentz

The Pressure Anomaly

Ref.: Estrada et al., *J. Phys. A* **45** (2012) 455402 (and 2009 and 2011 QFExt reports)

Properties of the pressure anomaly

1. Appears in regularized divergent terms.
2. Does not affect the finite (cutoff-independent) terms, or their logarithmic partners.
3. Can be remedied by improved regularization procedures.

For scalar field with orthogonally intersecting plates, the unregularized T_{00} and T_{11} pointwise satisfy the expected relation, but their totals diverge.

The standard ultraviolet regularization (point-splitting in imaginary time) violates the expected relation.

Point-splitting in a neutral spatial direction restores it. Pauli–Villars regularization (Lorentz-invariant) should also work. (See Mera's talk.)

The study of the coupling of plane boundary energy to the gravitational field (EFM paper and 2007 QFExt) should be extended to full Einstein equations with back reaction (Deepak Bastola's project).

With luck, that can be extended to spherical and cylindrical boundaries. (Recall that the anomaly first turned up in spherical *Mma* calculations by Schaden and Fulling.)

The Torque Anomaly

Refs.:

- Fulling et al., *Phys. Rev. D* **87** (2013) 047702 (and C. Trendafilova's senior research thesis)
- Dowker, arXiv:1302:1445
- Milton et al., in preparation
- Fulling et al., in progress

Properties of the torque anomaly

1. Appears in cutoff-independent terms (Deutsch–Candelas formula).

2. Probably does not affect cutoff-dependent terms.

3. Almost certainly stems from neglect of some physics near the axis.

4. Occurs also for the cone (Dowker).

In a wedge, if $\xi = \frac{1}{6}$ (or for EM field) the unregularized T_{00} and T_{11} are already finite on the sides (but diverge on the axis). They pointwise violate the expected relation between torque and the angular derivative of energy.

I am not sure whether the local torque anomaly infects the divergent terms when $\xi \neq \frac{1}{6}$. Detailed study of that case has been postponed until the Deutsch–Candelas case is thoroughly understood.

This anomaly has consistent sign, so the only visible resolution is that there is a compensating effect in the region around the axis. This is quite plausible, because in the truncated model (annular sector) the energy density and pressure along the inner cylinder probably do not go to 0 as the radius does.

Investigation of the truncated model is complicated by logarithmic divergences in the stress tensor along the curved boundaries.

Conclusion

These two anomalies are totally different things. They have different character and different origin.

The pressure anomaly is an artifact of a regularization procedure. More covariant regularizations appear to cure it.

The torque anomaly has nothing to do with regularization. It indicates that some physics has been left out of the calculation.

Puzzle: Why does ultraviolet regularization fail to remove the singularity at the axis, even when the angle is $\pi/2$? This demands reexamination.

Another agenda item: Study "conformal correction" term with cutoff included.