

The following sums are guaranteed to be equal:

**Eigenfunction sum:**

$$\frac{1}{2\pi} \sum_{k=-\infty}^{\infty} e^{ikx} e^{-ik^2t}$$

**Image sum:**

$$\frac{1}{\sqrt{4\pi it}} \sum_{n=-\infty}^{\infty} e^{i(x-2\pi n)^2/4t}$$

(What these functions have to do with the Schrödinger equation will be explained early in our course.)

To evaluate the sums numerically, we have to replace  $\pm\infty$  with some large integers  $\pm N$ . We plot both sums and increase  $N$  until the two curves almost agree, to be sure that we have taken enough terms.

Also, to make sure that the sums converge, we give  $t$  a small negative imaginary piece,  $-ip$ . This makes the terms in the sum decrease exponentially at large  $k$  and  $n$ . We are interested in the limit when  $p \rightarrow 0$ .

We will plot just the **real part** of the function, which reduces to the cosine sums on the Web page. The first part of the Maple session holds  $t$  fixed at various values and plots the resulting function of  $x$ . The second part of the Maple session (yielding the graphs on the Web page) sets  $x = 0$  and plots the resulting function of the real part of  $t$ , with the imaginary part (the cutoff parameter) fixed at some small negative value.