List of topics discussed in Differential Geometry-2, MATH 623

1. (some of this may be covered at the end of DG-1 but will be repeated here in depth) The concepts of vector bundles, connections on vector bundles, the curvature tensor of a connection, the connection and curvature form of a connection in local trivialization, parallel transport, holonomy, Ambrose -Singer theorem (formulation and discussion), geodesics. (Differential) Bianchi identity.

2. Connections on tangent bundle, its torsion tensor and form, the algebraic Bianchi identity, Riemannian manifolds and Levi-Civita connection. Algebraic properties of Riemannian curvature tensor, Ricci and scalar curvature, sectional curvature and its geometric interpretation, Weyl conformal tensor. Brief discussion of spin structures and the Young-Mills functional.

3. Basics on Lie groups and Lie algebra: a notion of Lie groups and its Lie algebra, exponential map, a construction of Lie group by its Lie algebra (brief), Lie subgroup, adjoint representations, Killing form, bi-invariant metrics on Lie groups and their Riemannian curvature tensor, elements of homogeneous spaces and symmetric spaces; Maurer-Cartan form on a Lie group.

4. Geometry of submanifolds in homogeneous spaces as an important application of the Frobenius theorem via the Maurer-Cartan form on a Lie group, the notion of the Darboux derivative of a map from a manifold to a Lie group and the fundamental theorem for Darboux derivatives via the Cartan's graph method. The Geometry of submanifolds of Riemannian manifolds: the second fundamental form, the Gauss, Ricci, and Codazzi equations, the fundamental theorem of submanifolds in $\mathbb{R}^n$.

5. Elements of calculus on Riemannian manifolds: Hodge star operator, gradient, divergence, Laplacian of differential forms and harmonic forms, basic on De Rham cohomology and Hodge theorem (formulation and discussions only).

6. Variational theory of Riemannian geodesics: the energy functional and its first variation, geodesics as criticial points, the second variation along geodesics, Jacobi fields, conjugate points, the Morse index theorem (formulation and discussions, the proof is a topic for final presentation together with applications in homotopy theory via Morse theory); expansion of volumes and geometric interpretation of Ricci and scalar curvature. Bonnet-Myers theorem and its consequences for the topology of a manifold. Other types of comparison theorems like the Rauch and the volume comparison theorem are usually left for final presentations,

7. Elements of the theory of principle connections: definition, curvature form and its property.

8. Overview of Chern-Weil theory of characteristic classes: invariant polynomials, Chern and Pontryagin classes, the Euler class, the Gauss-Bonnet-Chern theorem.

9. If time permits I also discuss G-structures and their prolongations, elements of Cartan connection, with applications to Riemannian and conformal geometry, as this topic is more close to my own research.