Weekly schedule for MATH308H Fall 2021

(introducing systems of ODE's as early as possible)

Week 1 *First order differential equations with explicit solution*

(a) Equations of the form $\frac{dy}{dx} = f(x)$, $\frac{dy}{dx} = f(y)$ (section 1.2), separable equations (section 2.2); **Enrichment :** Examples of equations that are initially not separable but can be reduced to separable.

(b) First order linear nonhomogeneous system (section 2.1) including example of mixing problem from section 2.3; **Enrichment:** Bernoulli's equations

(c) Exact equations (section 2.6).

Week 2 (a) Geometric interpretation of solution of first order equations and systems: direction fields for non-autonomous equations and systems and vector fields for autonomous equations/systems (section 1.1 and 7.1). Very briefly on the Euler method in relation to the direction field approach (section 2.7).

(b) Informal discussion of existence and uniqueness theorems (following section 2.4) both for equations and systems including:

i. Back to the Euler method as intuitive explanation for existence of solutions of IVP with continuous right hand-side (section 2.7).

ii. An example of IVP for which uniqueness fails;

iii. An example of IVP for which solutions blow up to infinity in finite time;

iv. Formulation and discussion of existence and uniqueness theorem for arbitrary first order equations/ system including the discussion of importance of the assumptions and conclusions of this theorem in light of examples in items ii and iii above;

v. Definition of linear homogeneous equations of arbitrary order and linear system of first order and how to transform the equation of higher order to a system of first order (as discussed in section 7.1);

vi. Formulation and discussion of existence and uniqueness theorem for linear equations and systems with emphasis on more strong conclusion (absence of blow-up in finite time) compared to the general theorem.

(c) Qualitative analysis of first order autonomous equations: phase line, stability of critical points (the role of linearization near the critical point in the question of stability can be already emphasized here). Applications to the model of population dynamics.

Week 3 General principles of linear homogeneous equations of arbitrary order and linear system of first order including:

(a) Superposition principle (as in sections 3.2 and 7.4 combined)

(b) Some basic facts on matrices (section 7.2), including

    i. matrix multiplication and matrix representation of linear systems of first order;

    ii. determinants and linear dependence/independence

(c) Fundamental set of solutions and the Wronskian both for linear equations of higher order and linear systems (sections 7.4 and 3.2 combined);

(d) Eigenvalues and eigenvectors and their role in systems of differential equations with constant coefficients. The case of distinct real eigenvalues (section 7.3, 7.5).

Week 4 (a) Applications to second order linear homogeneous equations with constant coefficients: the cases of distinct real roots of characteristic polynomial (sections 3.1).

(b) Complex eigenvalues (section 7.6) and application to second order linear homogeneous equations with constant coefficients: the case of complex roots (section 3.3) with applications to undamped an damped vibrations (section 3.7).

Week 5 (a) **Midterm exam I**

(b) Fundamental matrices , matrix exponential (section 7.7);

(c) Repeated eigenvalues: algebraic and geometric multiplicity of an eigenvalue, the case when geometric multiplicity is equal to algebraic multiplicity for each eigenvalue (end of section 7.5).

Week 6 (a) Repeated eigenvalues: the case when geometric multiplicity is smaller than the algebraic multiplicity for some eigenvalues, the notion of generalized eigenvectors and calculation of fundamental set of solutions using the basis of generalized eigenvectors (section 7.8);

(b) Second order ODE, the case of repeated roots (section 3.4). **Enrichment:** Euler's equations (Beginning of section 5.4 and problem 34 , page 166 of the textbook)

Week 7 (a) Non-homogeneous linear equations and linear systems of equations. Method of variation of parameters for linear systems of equations with applications to higher order scalar linear equations (section 3.6 and section 7.9 combined).

(b) Method of undetermined coefficients for second order ODE (section 3.5); Applications to forced vibrations (section 3.8).

Week 8 (a) The Phase Plane; Linear Systems, classification of critical points (section 9.1)

(b) Autonomous Systems, Critical points, and Stability (section 9.2);

Week 9 (a) Main properties of Laplace transform (section 6.1).

(b) Solution of initial value problems using Laplace transform (section 6.2 and the part of section 7.9 concerning Laplace transform).

Week 10 (a) **Midterm exam II**

(b) Step functions. Differential equations with discontinuous forcing functions (sections 6.3 and 6.4);

(c) Impulse functions (section 6.5); The convolution integral (section 6.6).

Week 11 Locally linear systems (section 9.3) with application to competing species (section 9.4) and predator-prey model (section 9.5).

Week 12 Basic series solutions (sections 5.1, 5.2, 5.3).

Week 13, 14 (a) More examples of nonlinear systems and Lyapunov method (as in section 9.6).

(b) Basic numerical methods (sections 8.1, 8.2, 8.3).