Workshop on "Geometry of vector distributions, differential equations, and variational problems"

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Abstracts of the talks

Jan Slovak (Masaryk University in Brno, Czech Republic)

Inclusions of parabolic geometries on a manifold

The lecture reports on joint research with Boris Doubrov aiming at the classification and study of all Fefferman type constructions for parabolic geometries where the underlying manifolds do not change.

The original Fefferman's construction produces an S^1–bundle over each manifold with an integrable CR–structure, equipped with a conformal structure. This construction corresponds to the embedding of Lie algebras $\mathfrak{g} = \mathfrak{su}(p+1, q+1)$ into $\tilde{\mathfrak{g}} = \mathfrak{so}(2p+2, 2q+2)$. Recently, two constructions appeared which may be interpreted as instances of such a procedure with the property that the underlying manifold does not change. Indeed conformal structures naturally associated with non-degenerate rank 2 vector distributions on 5-dimensional manifolds were studied by Nurowski and those associated with non-degenerate rank 3 distributions on 6-dimensional manifolds by Bryant. In both cases there are natural parabolic geometries associated with these distributions, which serve as an intermediate structure between the distribution and the conformal geometry.

Using classical results by A. Onischchik, we classify all possibilities of such inclusions of parabolic geometries. Apart of known examples, a new series of embeddings of 2–graded C_ℓ geometries into 1–graded $D_{\ell+1}$ geometries has been detected. These geometries correspond to the generic ℓ–dimensional distributions of codimension $\frac{1}{2}\ell(\ell-1)$ and the Bryant's example fits into this series with $\ell = 3$.