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Abstracts of the posters

An estimate for the entropy of Hamiltonian flows

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We present a generalization to Hamiltonian flows on symplectic manifolds of the estimate proved by Ballmann and Wojtkovski for the dynamical entropy of the geodesic flow on a compact Riemannian manifold of nonpositive sectional curvature. Given such a Riemannian manifold M, Ballmann and Wojtkovski proved that the dynamical entropy h_μ of the geodesic flow on M satisfies the following inequality:

$$h_\mu \geq \int_{SM} \mathrm{Tr}\sqrt{-K(v)}\, d\mu(v),$$

where v is a unit vector in T_pM, if p is a point in M, SM is the unit tangent bundle on M, $K(v)$ is defined as $K(v) = \mathcal{R}(\cdot, v)v$, with $\mathcal{R}$ Riemannian curvature of M, and μ is the normalized Liouville measure on SM.

We consider a symplectic manifold M of dimension $2n$, and a compact submanifold N of M, given by the regular level set of a Hamiltonian function on M; moreover we consider a smooth Lagrangian distribution on N, and we assume that the reduced curvature $\hat{R}^h_z$ of the Hamiltonian vector field $\vec{h}$ with respect to the distribution is nonpositive. Then we prove that under these assumptions the dynamical entropy h_μ of the Hamiltonian flow w.r.t. the normalized Liouville measure on N satisfies:

$$h_\mu \geq \int_N \mathrm{Tr}\sqrt{-\hat{R}^h_z}\, d\mu.$$